

# Goodput and Channel Allocation in Opportunistic Spectrum Access Networks

Waqas Ahmed\*, Jason Gao\*, Hong Zhou<sup>†</sup> and Michael Faulkner\*

\* Centre for Telecommunications and Microelectronics (T $\mu$ E), Victoria University

PO Box 14428, Melbourne, Victoria 8001, Australia

<sup>†</sup>Faculty of Engineering & Surveying, University of Southern Queensland

E-mail: waqas.ahmed@live.vu.edu.au, hzhou@usq.edu.au, {jason.gao, mike.faulkner}@vu.edu.au

**Abstract**—In this paper, the expressions for the forced termination probability, blocking probability and aggregate goodput of a single channel SU network with fixed packet size are derived. For a given SU data rate and fixed header length, it has been shown that there exists an optimal payload length for SUs packets which maximizes the aggregate goodput. Using the derived results, the impact on the aggregate goodput by three channel allocation methods i.e., intuitive, optimal, and proposed heuristic is investigated in a multiple channel CR network. Numerical studies show that the goodput of the proposed heuristic method is about 90-95% of that of the optimal method, and about 20-25% better than that of the intuitive method. Simulation results are also included.

## I. INTRODUCTION

Years of growth in wireless communication services and conservative spectrum allocation policies have created a shortage of vacant spectrum bands (channels). On the other hand, experimental studies [1] [2] have shown that the utilization of licensed (primary) frequency channels is generally quite low. It is clear that the future growth of wireless communications is very much dependent on increasing the efficiency of primary spectrum.

Cognitive radio [3] also known as opportunistic spectrum access (OSA) has emerged as a promising solution to increase the spectrum efficiency [4]. Cognitive radio enables the unlicensed (secondary) users to opportunistically build transmission links using vacant primary channels. However, to avoid interference to primary users (PUs) the secondary users (SUs) must immediately vacate these channels when PUs reappear. Due to the arrival of a PU, an active SU connection (packet) is terminated prematurely. This is referred to as the forced termination. The blocking of a new SU packet occurs when there is no vacant channel.

To understand the system wide benefits of the CR networks, various authors studied the performance of a SU stream in terms of quality of service i.e, forced termination probability, blocking probability. In previous work, for a spectrum pooling system, the authors in [5] provide a numerical study of the forced termination and blocking probabilities experienced by SUs. It has been shown that bandwidth utilization increases when a primary network is aware of SUs existence and assign channels so as to avoid SUs termination. In [6], for an exponentially distributed packet length of SUs, authors analyzed the impact of spectrum handoff on forced termination probability,

blocking probability. However, the analysis presented in [5] [6] is restricted to an exponentially distributed SU connection length and doesn't consider the transmission overhead. Authors in [7] have proposed and analyzed random access schemes employing different sensing and backoff mechanisms for throughput maximization. However, the analysis is limited to saturated SU traffic conditions.

In context of allocation, the authors in [8] proposed cooperative and non cooperative channel allocation strategies based on color-sensitive graph coloring model. In [9], the authors proposed heuristic channel allocation algorithms based on multichannel contention graph and linear programming. In [10], authors derive the optimal access probabilities for fairness among two independent SUs in terms of throughput. The authors in [11] discussed the coexistence of dissimilar SUs in a cognitive radio network, using spectrum utilization as the benchmark. The techniques presented in [9] [10] [11] only discuss spectrum sharing without modelling the impact of SUs forced termination and blocking.

In this paper, firstly, using a continuous time Markov model the expressions of forced termination probability, blocking probability and aggregate goodput are derived for CR networks with a single channel and SU packet stream. An example of a practical single channel network is the current IEEE 802.11. For a fixed header length, it has been shown that there exists an optimal payload length which maximizes the aggregate goodput. We then apply these results to the CR networks with multiple channels and SU packet streams. The objective is to find the optimal channel allocation scheme for individual packet streams which maximizes the aggregate goodput. The optimal scheme is calculated by using an exhaustive search. In order to reduce the computational complexity of the search, a heuristic method is proposed based on an intuitive approach. Numerical results show that the proposed method has a significant improvement over the intuitive method, and closely follow the performance results of the optimal method.

The rest of the paper is organized as follows. In Section II we introduce the system model and key assumptions made throughout the paper. In Section III the mathematical analysis for the forced termination probability, blocking probability, goodput and optimal payload length of SU packets are provided. In Section IV, we investigate the optimal, intuitive and heuristic channel allocation methods for multiple SU packet streams in multi-channel CR networks. The numerical

investigations are carried out in Section V. Finally, the main findings are outlined in Section VI.

## II. SYSTEM MODEL

We consider a single channel secondary network overlaying a primary network using the same channel. The arrivals of new PU and SU packet streams follow an independent Poisson process denoted by  $\lambda_p$  and  $\lambda_s$ , respectively [5] [6]. It is assumed that the primary network is an M/M/1/1 loss network [12] in which the mean service rate of PUs packets is  $\mu_p$ . Within a SU stream each SU packet has a fixed header length  $\frac{1}{\mu_h}$  and the payload length  $\frac{1}{\mu_{fp}}$ . We assume that SU perfectly sense the status of the channel. The overlap time of PU and SU packets is not modelled and on the arrival of a PU packet the SU on the channel is terminated without any delay.

The system state is represented by  $(i, j, k)$ , where  $j(i) \in \{0, 1\}$  represents the status of the PU (SU) packets on the channel, i.e., the value of 0/1 shows the absence/presence of a user packet. When  $i = 1$ ,  $k \in \{1, \dots, K\}$  in system state represents the phase of a SU packet. The details on the concept of the phase representation is explained in the next section.

## III. SINGLE CHANNEL AND SINGLE SU PACKET STREAM

In this section we calculate the forced termination probability, blocking probability and goodput of SU packets with fixed payload length. The total SU packet length can be expressed by the sum of overhead and the payload lengths, i.e.,

$$\frac{1}{\mu_f} = \frac{1}{\mu_h} + \frac{1}{\mu_{fp}} \quad (1)$$

Due to the presence of fixed length SU packets, the system is not purely Markovian and cannot be represented by a simple birth-death process [13]. Here, we adopt the approach as in [12] to model the fixed length SU packet by a Markov chain. In this approach, each SU packet is represented by a continuous stream of  $K$  exponentially distributed sub-packets with independent and identical distributions. Note that the total length of these  $K$  i.i.d sub-packets is Erlang distributed and the individual sub-packet has an average length of  $\frac{1}{K\mu_f}$ . It is well known [12] that as  $K \rightarrow \infty$  the total length of all sub-packets is same as that of the fixed length SU packet  $\frac{1}{\mu_f}$ .

In the  $K$ -node Markov chain representation, the  $k^{th}$  state models the arrival of the  $k^{th}$  sub-packet. We call these states  $k \in \{1, \dots, K\}$  as the phases of a fixed length SU packet. Fig. 1 shows the complete system Markov chain model with the states represented by  $(i, j, k)$ . Note that the states inside the dotted line models the fixed length SU packet. The total mean departure rate from each phase of SU packet is  $(K\mu_f + \lambda_p)$  where  $K\mu_f$  is the completion rate of each i.i.d sub-packet and  $\lambda_p$  is the termination rate due to the arrival of a PU packet. A set of balance equation of the system Markov chain (Fig. 1)

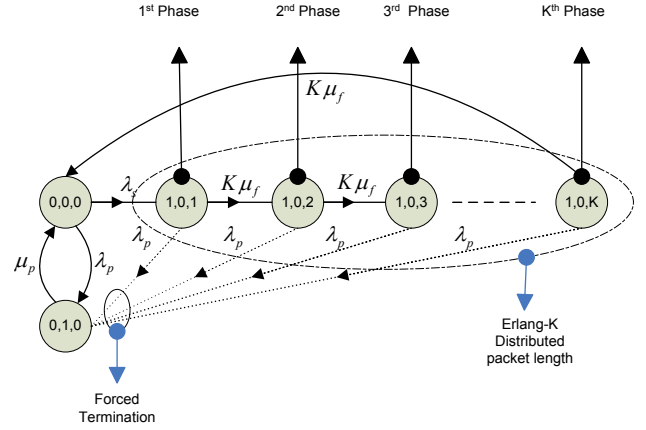


Fig. 1. System Markov chain model of SU packets

is shown as follows.

$$\begin{cases} (\lambda_s + \lambda_p)P(0, 0, 0) = \mu_p P(0, 1, 0) + K\mu_f P(1, 0, K) \\ (K\mu_f + \lambda_p)P(1, 0, 1) = \lambda_s P(0, 0, 0) \\ (K\mu_f + \lambda_p)P(1, 0, 2) = K\mu_f P(1, 0, 1) \\ \vdots \\ (K\mu_f + \lambda_p)P(1, 0, K) = K\mu_f P(1, 0, K-1) \end{cases} \quad (2)$$

The set of balance equations in (2) represents the equilibrium point of the system Markov chain. The state probabilities  $P(i, j, k)$  at an equilibrium point are calculated by solving (2) under the following fundamental constraint i.e.,

$$P(0, 0, 0) + P(0, 1, 0) + \sum_{k=1}^K P(1, 0, k) = 1 \quad (3)$$

Using state probabilities  $P(i, j, k)$ , the expressions for forced termination probability, blocking probability and goodput are derived below.

### A. Blocking probability

In Fig. 1, an incoming SU packet is blocked when the channel is occupied by either a PU or another SU packet. The blocking probability  $P_B^{Kf}$  in this scenario can be expressed as

$$P_B^{Kf} = 1 - P(0, 0, 0) \quad (4)$$

The blocking probability of a fixed length SU  $P_B$  packet can be calculated by taking the limits i.e.,  $P_B = \lim_{K \rightarrow \infty} P_B^{Kf}$ . After some simple algebraic manipulations of (2), (4) and using the following well known identity

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n, \quad (5)$$

the  $P_B$  becomes

$$P_B = 1 - \frac{\mu_p \lambda_p}{(\lambda_p + \mu_p) \left( \lambda_s + \lambda_p - e^{-\frac{\lambda_p}{\mu_f}} \lambda_s \right)} \quad (6)$$

### B. Forced termination probability

Forced termination probability is defined as the probability that an ongoing SU packet is terminated prematurely. In an equilibrium status, the sum of forced and unforced termination rate of SU packets on the channel must equal to the admission rate which is defined as the incoming rate minus the blocking rate. Therefore, the forced termination probability can be defined as follows

$$P_F^{Kf} = \frac{\text{SU forced termination rate}}{\text{SU Admission Rate}} \quad (7)$$

From Fig. 1,  $\sum_{k=1}^K P(1, 0, k)\lambda_p$  is the total forced termination rate of SU packets and  $(1 - P_B^{Kf})\lambda_s$  is the admission rate. Using the above definition, the forced termination probability of SU with Erlang- $K$  distributed packet length can be written as

$$P_F^{Kf} = \frac{\sum_{k=1}^K P(1, 0, k)\lambda_p}{(1 - P_B^{Kf})\lambda_s} \quad (8)$$

Using (2) and (4), the above equation can be expressed as

$$P_F^{Kf} = \frac{\lambda_p}{K\mu_f} \sum_{k=1}^K \left( \frac{K\mu_f}{K\mu_f + \lambda_p} \right)^k \quad (9)$$

The forced termination probability  $P_F$  of SU packets with fixed length can be derived from (9). This is achieved by first calculating the sum of the geometric series and thereafter taking the limits  $P_F = \lim_{K \rightarrow \infty} P_F^{Kf}$ . The final expression of  $P_F$  is given below

$$P_F = 1 - e^{-\frac{\lambda_p}{\mu_f}} \quad (10)$$

### C. Goodput

Assuming the normalized data rate  $R = 1$  bits per unit time. The aggregate network goodput is defined as the product of the number of completed SU packets per unit time and the average duration of the completed packets. Given the arrival rate  $\lambda_s$ , the SU packet completion rate is given by  $(1 - P_B)(1 - P_F)\lambda_s$ . The average payload duration of the completed SU packets is  $\frac{1}{\mu_{fp}}$ . Therefore, the aggregate payload goodput  $\rho$  can be written as

$$\rho = (1 - P_B)(1 - P_F)\theta_s \quad (11)$$

where  $\theta_s = \frac{\lambda_s}{\mu_{fp}}$  is the data rate of the incoming SU packets without overhead.

### D. Optimal SU Payload Length

In this section, we investigate the optimal payload length of SU packets which maximizes the aggregate goodput for a fixed header length and data rate. Mathematically, the optimization problem can be expressed by

$$\begin{aligned} \mu_{fp}^* &= \arg \max_{\mu_{fp}} \{\rho\} \\ \text{s.t.} \quad &\frac{\lambda_s}{\mu_{fp}} = \theta_s, \frac{1}{\mu_h} = c \end{aligned} \quad (12)$$

By substituting  $\lambda_s = \theta_s \mu_{fp}$  and  $\frac{1}{\mu_h} = c$  in (11), the critical points of the above optimization problem can be calculated by  $\frac{d}{d\mu_{fp}} \rho = 0$ . From this, the optimal fixed payload length  $\mu_{fp}^*$  of SU packets must satisfy the following equation.

$$\left( \frac{\lambda_p}{\mu_{fp}^* \sqrt{\theta_s}} \right)^2 + \frac{\lambda_p}{\mu_{fp}^*} + e^{-\lambda_p(c + \frac{1}{\mu_{fp}^*})} = 1 \quad (13)$$

The root of equation (13) can be calculated numerically using the standard techniques in [14]. Note that the mean optimal payload length  $\frac{1}{\mu_{fp}^*}$  must always be less than the mean interarrival length  $\frac{1}{\lambda_p}$  of PU packets as the right hand side of the (13) requires the middle term on the left  $\frac{\lambda_p}{\mu_{fp}^*} < 1$ .

## IV. MULTIPLE CHANNELS AND MULTIPLE SU PACKET STREAMS

In this section we investigate the channel allocation for multiple SU packet streams in a multi-channel CR network. Specifically, the objective is to allocate the number of SU streams to each channel such that the sum of the goodputs from all channels is maximized.

Let  $N$  be the total number of SU streams  $I \in \{1, \dots, N\}$  in  $M$  PU channels  $J \in \{1, \dots, M\}$ . We assume these streams are i.i.d. with the same mean arrival rate  $\lambda_s$  and service rates  $\mu_s$ . The PU arrival and service rates are  $\{\lambda_p^1, \dots, \lambda_p^M\}$  and  $\{\mu_p^1, \dots, \mu_p^M\}$ . Now let  $N_j$  denotes the number SU streams on channel  $j$ , then using the sum property of Poisson process [12] the arrival rate of combined SU stream on channel  $j$  is  $N_j \lambda_s$ . From (6), (10) and (11) the goodput  $\rho^j$  on channel  $j$  is given by following expression.

$$\rho^j = \frac{\mu_p^j \lambda_p^j e^{-\frac{\lambda_p^j}{\mu_p^j}} N_j \lambda_s}{\mu_s (\lambda_p^j + \mu_p^j) \left( N_j \lambda_s + \lambda_p^j - e^{-\frac{\lambda_p^j}{\mu_p^j}} N_j \lambda_s \right)} \quad (14)$$

To find the optimal channel allocation which maximizes the sum goodput, the objective function can be written as

$$\begin{aligned} N_j^* &= \arg \max_{N_j} \sum_{j=1}^M \rho_j, \\ \text{and} \quad &\sum_{j=0}^M N_j = N \end{aligned} \quad (15)$$

The optimal solution  $\{N_j^*\}$  can be easily obtained by an exhaustive search method.

### A. Heuristic Approach

In order to reduce the computational complexity of the above exhaustive search method, we propose a heuristic channel allocation method. In this method, we consider two key factors which affect the goodput on individual channels. The first is the *average channel opportunity* and the second is the *completion factor* of SU packets. The average channel opportunity  $O_j$  is given as

$$O_j = 1 - \frac{\lambda_p^j}{\lambda_p^j + \mu_p^j} \quad (16)$$

where  $\frac{\lambda_p^j}{\lambda_p^j + \mu_p^j}$  is the PU occupancy on channel  $j$ . Since the completion probability of SU packets on channel  $j$  is proportional to  $\frac{\mu_s}{\lambda_j}$  as shown in (10), the completion factor is given by

$$C_j = \frac{\mu_s}{\lambda_p^j} \quad (17)$$

The number of SU streams allocated to channel  $j$  needs to be proportional to  $O_j C_j$ , while satisfying the constraint  $\sum_{j=0}^M N_j = N$ . Therefore,  $N_j$  has the following form

$$N_j = \left\lfloor \frac{O_j C_j N}{\sum_{j=1}^M O_j C_j} \right\rfloor \quad (18)$$

where  $\lfloor \cdot \rfloor$  indicates the rounding to nearest integer.

## V. NUMERICAL RESULTS

### A. Single channel and Single SU packet stream

In this section, we investigate the forced termination probability  $P_F$ , blocking probability  $P_B$ , and goodput  $\rho$  in a single channel CR network. In all the examples, we set  $\lambda_p = 1$ ,  $\mu_p = 4$  which gives a PU channel occupancy of 20%. In computer simulations, the following methods are used to compute the forced termination probability  $P_F$ , blocking probability  $P_F$  and aggregate goodput  $\rho$ .

$$P_F = \lim_{T \rightarrow \infty} \frac{\text{Total No. of terminated packets in } [0, T]}{\text{Total No. of Admitted packets in } [0, T]}$$

$$P_B = \lim_{T \rightarrow \infty} \frac{\text{Total No. of blocked packets in } [0, T]}{\text{Total No. of Generated packets in } [0, T]}$$

$$\rho = \lim_{T \rightarrow \infty} \frac{\text{Total Payload length of completed packets } [0, T]}{T}$$

In Fig. 2 the blocking probability  $P_B$ , forced termination probability  $P_F$  are plotted against the mean payload length  $\frac{1}{\mu_{fp}}$ , for an incoming data rate  $\theta_s = 1$ . In the figure,  $P_F$  is an increasing function whereas  $P_B$  is a decreasing function. This is expected, because for the forced termination probability, longer the packet length the more chances of collision with PU packets. For the blocking probability, due to the fixed payload intensity, smaller packet lengths means faster arrival rates, therefore more chances that SU packets are blocked.

Fig. 3 shows the goodput curves  $\rho$  of the SU packets under the same condition as above. The figure shows that there exists an optimal payload length which maximizes the goodput. The reason is that the goodput is a function of blocking and forced termination probabilities, and the duration of the payload. For a given data rate, the higher the payload length, the smaller the blocking probability, the more efficient is the transmission per packet. On the other hand, large packet size results in higher forced termination probability, hence reduced goodput. Therefore, combining all these factors, there is an optimal tradeoff which maximizes the goodput.

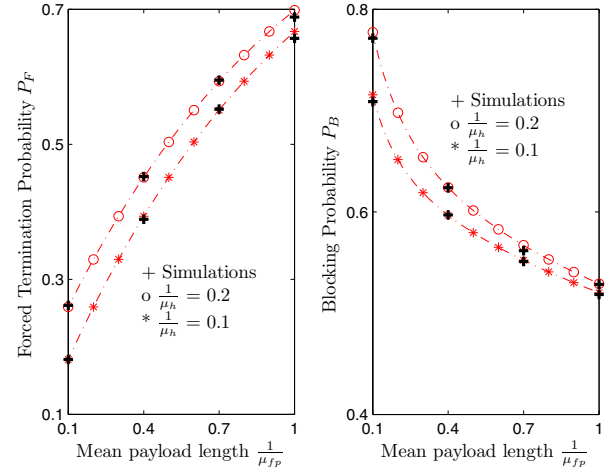


Fig. 2. Left: Forced termination probability  $P_F$ , Right: Blocking probability  $P_B$  versus SU mean payload length  $\frac{1}{\mu_{fp}}$  given  $\{\lambda_p, \mu_p\} = \{1, 4\}, \theta_s = 1$

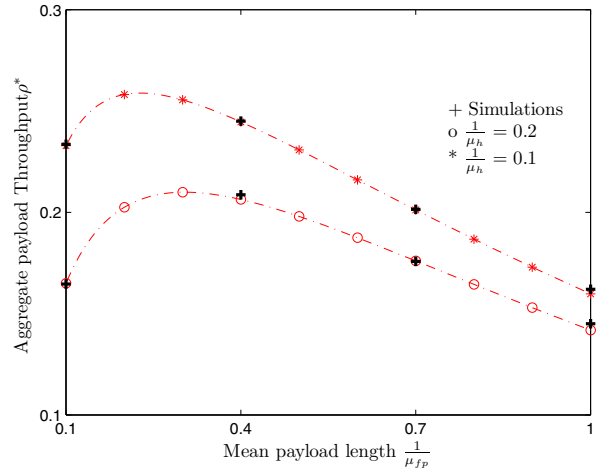


Fig. 3. Aggregate payload throughput of SU packets  $\rho$  versus mean payload length  $\frac{1}{\mu_{fp}}$  (given  $\{\lambda_p, \mu_p\} = \{1, 4\}, \theta_s = 1$ )

### B. Multiple Channel and Multiple SU packet streams

In this subsection we compare the results of optimal allocation with an intuitive method and the proposed heuristic allocation method. The intuitive method is based on the belief that the number of SU streams on the channel is proportional to the average channel opportunities, i.e.,

$$N_j = \lfloor O_j N \rfloor \quad (19)$$

For the numerical example we choose the number of PU channels  $M = 3$ , the arrival rate and service rate of each SU packet stream  $\lambda_s = 0.2$  and  $\mu_{fp} = 1.5$ , respectively. The header length of each packet is fixed  $\frac{1}{\mu_h} = 6$ . The PU arrival rate and service rates are  $\{\lambda_p^1, \lambda_p^2, \lambda_p^3\} = \{0.9, 1.5, 3\}$  and  $\{\mu_p^1, \mu_p^2, \mu_p^3\} = \{0.9, 3.5, 12\}$  which corresponds to the PU channel occupancy of  $\{0.5, 0.3, 0.2\}$ .

In Fig. 4, the sum goodput of all channels is plotted against the total number of SU streams  $N$ . The differences between the proposed and intuitive methods are about 20-25%, whereas

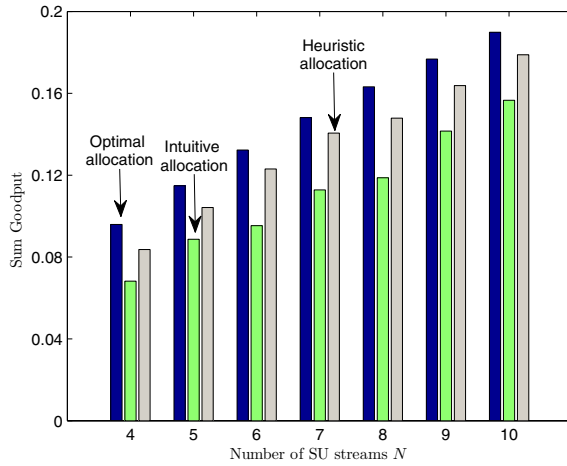


Fig. 4. Sum goodput of SU packet streams given  $M = 3$ ,  $\lambda_s = 0.2$ ,  $\mu_{fp} = 1.5$ ,  $\mu_h = 6$   $\{\lambda_p^1, \lambda_p^2, \lambda_p^3\} = \{0.9, 1.5, 3\}$ ,  $\{\mu_p^1, \mu_p^2, \mu_p^3\} = \{0.9, 3.5, 12\}$

the differences between the proposed and optimal methods are about 5-10%.

## VI. CONCLUSION

In this paper, using a continuous time Markov model, we have presented the exact solutions for the forced termination and blocking probabilities, and aggregate goodput for a single channel CR network overlaying a primary network. For a given data rate and fixed header length it has been shown that there exists an optimal payload length which maximizes the aggregate goodput. In multiple channel CR networks we proposed a low complexity heuristic channel allocation method. Numerical results show that proposed method achieves about 20-25% performance improvement over the intuitive method while trailing the optimal by about 5-10%.

## ACKNOWLEDGMENT

This research is supported under the Australian Research Council's Discovery funding scheme (DP0774689).

## REFERENCES

- [1] Federal Communications Commission (FCC), "Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies," ET Docket No. 03-108, Mar. 2005.
- [2] M. H. Islam *et al.*, "Spectrum survey in Singapore: Occupancy measurements and analyses," in *Proc. CROWNCOM 2008*, Marina Bay, Singapore, May 2008.
- [3] J. Mitola III, *Cognitive radio: An integrated agent architecture for software defined radio*, Ph.D. Thesis, KTH Royal Institute of Technology, Sweden, May 2000.
- [4] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol. 23, pp. 201-220, Feb. 2005.
- [5] F. Capar, I. Martoyo, T. Weiss and F. Jondral, Comparison of bandwidth utilization for controlled and uncontrolled channel assignment in a spectrum spooling system, in *Proc. IEEE VTC Spring 2002*, Birmingham, AL, May 2002, pp. 1069-1073.
- [6] Y. Zhang, "Dynamic Spectrum Access in Cognitive Radio Wireless Networks," in *Proc. IEEE ICC 2008*, Beijing, China, May 2008, pp. 4927-4932.
- [7] S. Huang, X. Liu, Z. Ding, "Opportunistic Spectrum Access in Cognitive Radio Networks," in *Proc. IEEE INFOCOM 2008*, Phoenix, AZ, April 2008, pp. 1427-1435.

- [8] H. Zheng and C. Peng, "Collaboration and fairness in opportunistic spectrum access," in *Proc. IEEE ICC 2005*, May 2005, pp. 3132-3136.
- [9] J. Tang, S. Misra, G. Xue, "Joint spectrum allocation and scheduling for fair spectrum sharing in cognitive radio wireless networks," *Computer Networks*, vol. 52, 2008.
- [10] B. Wang, Z. Ji, and K. J. R. Liu, "Primary-Prioritized Markov Approach for Dynamic Spectrum Access," in *Proc. of IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN) 2007*, pp. 507-515, Dublin, Ireland, Apr. 2007.
- [11] Arslan, Hseyin (Ed.), *Cognitive Radio, Software Defined Radio, and Adaptive Wireless Systems*, Signals and Communication Technology. Springer, 2007.
- [12] J. Mehdi, *Stochastic models in queueing theory*, 2nd ed., Academic Press, Oct. 2002.
- [13] G. Bolch, S. Greiner, H.-D. Meer, K. S. Trivedi, *Queueing Networks and Markov Chains: Modelling and Performance Evaluation with Computer Science Applications*, 2nd ed., Wiley-Interscience, 1998.
- [14] H.M. Antia, *Numerical Methods for Scientists and Engineers*, 2nd ed., Birkhuser Basel, 2002.